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ANALYSIS OF THE TWO-DIMENSIONAL STEADY-STATE BEHAVIOR OF EXTENSIBLE FREE-FLOATING CABLE SYSTEMS

by

Henry T. Wang and Thomas L. Moran

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An analysis is given for the two-dimensional steady-state behavior of extensible free-floating cable systems. The analysis includes the differential equations of equilibrium for an extensible cable and the iteration schemes for the equilibrium system drift velocity and surface buoy draft. Based on this analysis, a FORTRAN IV program, FF2E, has been written. The program allows the cable system to have an arbitrary number of different cable segments and intermediate bodies. The program works well for the large majority of cases of practical interest.

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NOTATION

B	Buoyancy
C_D	Drag coefficient
C_DA	Drag area
c	Current as a function of depth
D	Drag
d	Cable diameter
d_S	Diameter of the surface buoy
E	Resultant error $= \sqrt{e_H{}^2 + e_V{}^2}$
e	Strain $= (ds - ds_o)/ds_o$
e_H	Error in the horizontal direction $= T_H - (D_B + T_{BH})$
e_V	Error in the vertical direction $= T_V - (W_B + T_{BV})$
e_{VO}	Value of e_V when $e_H < e_1$
e_1	Small prescribed value
F	Force
f	Tangential drag coefficient
f_c	Function of strain $= d/d_o$
G	Drag force per unit length tangential to the cable
g	Gravity constant
H	Buoy draft
I	Drag force per unit length normal to the cable
K	Strain as a function of $(T - T_o)$
K_1	Compliance coefficient for a linearly elastic cable $= K/(T - T_o)$

k	Maximum perpendicular distance between the chord and lower array
ℓ	Cable length
M_U	Numerical factor = $\rho C_D A_T (c_M - c_m)/2$
M_V	Numerical factor = $\rho g \pi d_S^2/4$
N	Total number of cable segments = total number of bodies
p	Poisson's ratio
q	Fluid velocity relative to the cable = $c - U_D$
R	Drag force per unit length when the cable segment is normal to the stream = $\rho C_D d q\|q\|/2$
s	Distance measured along the cable
T	Cable tension
U_D	Drift velocity
W	Weight per unit length in water = $W_a - W_b$
W_a	Weight per unit length in air
W_b	Buoyancy force per unit length
W_B	Weight in water of the bottom unit
X	Direction normal to a cable segment
x	Horizontal direction positive to the right
Y	Direction tangential to a cable segment
y	Vertical direction positive downwards
δ	Positive quantity
Δ	Incremental value
ϵ_1	Small positive constant
ϵ_2	Small positive constant

Θ Angle with the vertical of a chord connecting the top of the array with the bottom weight

ρ Fluid mass density

ϕ Angle of the cable with the horizontal

ϕ_V Angle of the cable with the vertical

SUBSCRIPTS

a Above

b Below

B Bottom unit

H Horizontal direction

I Intermediate body

i Integer index

M Maximum

m Minimum

o Reference state

p Prescribed value

r Lower array

S Surface buoy

T Total

V Vertical direction

x x direction

y y direction

SUPERSCRIPTS

New values

ABSTRACT

An analysis is given for the two-dimensional steady-state behavior of extensible free-floating cable systems. The analysis includes the differential equations of equilibrium for an extensible cable and the iteration schemes for the equilibrium system drift velocity and surface buoy draft. Based on this analysis, a FORTRAN IV program, FF2E, has been written. The program allows the cable system to have an arbitrary number of different cable segments and intermediate bodies. The program works well for the large majority of cases of practical interest.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

Until recently, steady-state cable studies have been concerned largely with towing and single point moored cables. For cases of a cable towing a given body or of a single cable mooring a surface buoy with a given draft, the problem reduces to an initial value problem where the differential equations of equilibrium of the cable are integrated along the cable for given initial values at one end of the cable. The integration stops when cable scope or the prescribed depth is reached. With the use of subroutines for the solution of differential equations, such as the subroutine KUTMER which uses the Kutta-Merson method, these initial value problems can be conveniently solved on modern day digital computers. At NSRDC, Cuthill[1] has developed a program for the two-dimensional shape of inextensible towed and moored cables in a uniform stream. More recently, Wang[2] has developed a program for the three-dimensional shape of an extensible cable system moored in the presence of a current profile which may change in magnitude and direction with depth.

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[1]References are listed on page 23.

Perhaps the simplest case where iteration techniques are required in the steady-state analysis of cable systems is the case of a cable of given scope moored in a given depth. In this case the submergence of the surface buoy must be iterated until the cable reaches the given depth with the given cable scope. Where the current is severe and/or the depth is large, the buoy may be completely submerged beneath the ocean surface.

The use of iteration techniques is required in the steady-state analysis of some modern applications of cable systems. Skop and O'Hara[3] have studied the problem of subsurface buoys moored by an array of cables. The need for iteration procedures for this problem can be readily seen by considering the case of a single subsurface buoy moored by more than one cable. It is clear that the subsurface buoy can be maintained in equilibrium by an infinite combination of the forces in the cables just below the buoy. Conceptually, the forces at the tops of these cables must be continually iterated until the lower ends of these cables reach the required anchor positions. Instead of iterating the forces at the top of the cables, Skop and O'Hara choose the equivalent method of iterating the forces at the lower end, i.e., at the anchor positions.

In the case of a cable towed in a circular path the radius of the motion of the towed body must be continually iterated until the boundary conditions at the towing end are satisfied. This problem has been considered by Huang,[4] Choo,[5] Skop,[6] and Skop and Choo.[7]

In the case of free-floating cable systems, the problem considered in the present study, iteration schemes are also required. Basically, the drift velocity of the entire cable system and the draft of the surface buoy must be continually varied until the entire system is in an equilibrium of forces in both the horizontal and vertical directions.

The present report gives a description of computer program FF2E, which is a FORTRAN IV program for the two-dimensional steady-state behavior of extensible free-floating cable systems. The iteration schemes for the system drift velocity and buoy draft are described in detail. It is shown that the schemes work well for the large majority of cases of practical interest. For some cases of long cables in unusually severe current profiles the schemes may fail to converge due to the extreme sensitivity of cable configuration to changes in drift velocity and buoy draft. The

differential equations of equilibrium used in the present study are similar to, but not identical with, the equations used by Pode.[8] The differences are simply due to the difference in coordinate systems used. The program allows the cable system to have an arbitrary number of different cable segments and intermediate bodies. The surface buoy is taken to be a circular cylinder in the present study. However, by properly adjusting the values for the length, diameter, and drag coefficient, other buoy shapes may be approximated. For cases where the free-floating system contains a lower array of hydrophones, the program calculates the inclination and curvature of the array.

In the present study, the inclination of the buoy is taken to be vertical. In actual cases, the buoy inclination is a function of such factors as the center of gravity of the buoy, the center of buoyancy, the point of attachment of the cable to the buoy, and the magnitude and location of the resultant drag force (which is dependent on the inclination of the buoy) acting on the buoy. The inclination of the buoy must be determined by solving a transcendental equation for each new assumed set of values for drift velocity and buoy draft. This would result in increased complexity of program logic and computer time requirements. It is felt that the present simple approach for the surface buoy will yield, in most cases, reasonable answers for such system variables as system drift velocity, cable configuration, and operating depth of the bottom unit. The equilibrium inclination of the surface buoy can then be calculated from the equilibrium values of the drift velocity and buoy draft furnished by the present computer program. For those cases where the effect of surface buoy inclination substantially affects cable system variables, then this effect should, of course, be incorporated into the program.

The program may also be used to approximately solve cable payout problems. In these applications, an object may be kept stationary on the ocean bottom by having an intermediate cable pack pay out cable as it drifts along. The amount of cable which is payed out depends on the drift velocity of the cable pack. The actual drift of the cable pack is rather complex. The weight of the cable pack decreases as cable is being payed out, and the tension exerted on the cable pack by the cable being payed out is a function of time. An approximate analysis of the motion of the cable pack may

be made by assigning average values to the cable pack weight and the vertical and horizontal components of tension exerted on the cable pack by the cable being payed out. The resulting cable system consisting of surface float, connecting cable, and cable pack may then be considered as a free-floating system.

GENERAL METHOD OF SOLUTION

The free-floating cable system considered in the present study is shown in Figure 1. The system consists of a surface buoy, an arbitrary number of different cable segments and intermediate bodies, and a terminal weight at the bottom. As shown in Figure 1, a current profile which varies with depth may be prescribed over the entire cable system.

In the presence of the given current profile, the entire cable system will drift at a velocity U_D and the surface buoy will take a draft H such that the entire cable system is in an equilibrium of forces. The principal problem of the present study is to determine these equilibrium values of U_D and H. Using the iteration schemes described later in the study, values for U_D and H are chosen. From these values of U_D and H, the current relative to the cable system as well as the tension and angle of the cable just below the buoy are determined. The differential equations of equilibrium may then be integrated down the cable. At intermediate points along the cable the integration of the equations must be interrupted to take into account intermediate attached bodies which may be drogues or hydrophones.

The integration of the equations proceeds in the above manner until the bottom unit is reached. At this point, the equilibrium of the bottom unit is checked. Since the conditions at the top of the cable are determined by equilibrium considerations for the surface buoy, and the cable configuration is obtained by considering the equilibrium of the cable, then the entire system is in equilibrium if the bottom unit is in equilibrium. Figure 1 shows that the imbalance of forces in the horizontal and vertical directions may be written as

$$e_H = T_H - (D_B + T_{BH}) \tag{1}$$

$$e_V = T_V - (W_B + T_{BV}) \tag{2}$$

where e_H and e_V are the imbalances in the horizontal and vertical directions, respectively,

T_H and T_V are the horizontal and vertical components of the tension just above the bottom unit, respectively,

D_B and W_B are the drag and weight in water of the bottom unit, respectively, and

T_{BH} and T_{BV}, in the case of a payout problem, refer respectively to the horizontal and vertical components of tension exerted by the cable being paid out on the bottom unit; for free-floating systems, $T_{BH} = T_{BV} = 0$.

As mentioned previously, the quantities T_{BH} and T_{BV} are taken to be constants in the present study.

A number of criteria on the magnitude of the errors may be used to determine when the problem is considered to be solved. For example, one possible criterion would be to say that if the resultant imbalance in forces E, defined by

$$E = \sqrt{e_H^2 + e_V^2} \tag{3}$$

is less than a prescribed amount E_p, the problem is considered to be solved.[3] If $E > E_p$, then new values must be found for U_D and H. In the present study the problem is considered to be solved if the following two inequalities are simultaneously satisfied

$$|e_H| < \varepsilon_1 \, |D_B + T_{BH}| \tag{4a}$$

$$|e_V| < \varepsilon_2 \, |W_B + T_{BV}| \tag{4b}$$

where ε_1 and ε_2 are small positive constants.

EQUATIONS OF EQUILIBRIUM

Figure 2 shows the coordinate systems used in the present study. The (x,y) spatial coordinate system is taken to have fixed directions, with

the origin at the undisturbed ocean surface. The x axis is positive to the right and the y axis is positive downwards. The cable differential equations are written for a coordinate system attached to the cable, called the cable coordinate system (X,Y). The Y axis is directed along the cable.

CABLE EQUATIONS

Differential Equations

The two-dimensional differential equations of equilibrium for a flexible cable have been derived by a number of authors; see, for example, the derivation given by Pode.[8] For the coordinate systems used in the present study, the equations of equilibrium for the cable in an arbitrary stretched condition are

$$\frac{dT}{ds} + G + W \sin \phi = 0 \tag{5}$$

$$T \frac{d\phi}{ds} + I + W \cos \phi = 0 \tag{6}$$

where T is the tension,

s is the stretched cable scope,

W is the weight per unit length in water of the stretched cable, and

G, I are the fluid drag forces per unit length acting on the cable in the Y and X directions, respectively.

The above equations are similar to, though not identical with, the equations derived by Pode.[8] The differences are due to the difference in coordinate systems used. The present coordinate system defines the vertical coordinate as being positive downwards whereas in Reference 8 it is positive upwards. Also, for $0 \leq \phi \leq 90$ deg, cable scope in the present study is positive measured from the surface buoy, whereas in Reference 8 cable scope is positive measured from the bottom weight. The present coordinate system is more convenient for free-floating and moored cable systems while the coordinate system used in Reference 8 is more convenient for towing cables.

As pointed out in Reference 2, it is convenient to relate the quantities ds, G, I, and W to quantities at some reference state where all the

cable parameters are known. This reference state is taken to occur at $T = T_o$, where T_o need not be equal to zero. Summarizing the results of Reference 2, the relationships between ds, G, I, and W in terms of these quantities at the reference state $T = T_o$ are given by

$$ds = ds_o\ (1 + e) \tag{7}$$

$$I = f_c(e)\ I_o \tag{8a}$$

$$G = f_c(e)\ G_o \tag{8b}$$

$$W = \frac{W_{ao}}{1 + e} - W_{bo}\ [f_c(e)]^2 \tag{9}$$

where subscript o denotes quantities for the reference state,

e is the strain = $K(T - T_o)$, (10a)

K is an experimentally measured tension-strain function,

$$f_c(e) = \frac{d}{d_o} \tag{10b}$$

d is the cable diameter,

W_{ao} is the weight per unit length in air of the cable,

W_{bo} is the buoyancy force per unit length

$$W_{bo} = \rho\ g\ \frac{\pi d_o^{\ 2}}{4} \tag{10c}$$

ρ is the fluid density, and

g is the gravity constant = 32.2 ft/sec^2.

In the present study the cable is assumed to be linearly elastic so that K and f_c are given by

$$K = K_1 \cdot (T - T_o) \tag{11a}$$

$$f_c(e) = 1 - pe \tag{11b}$$

where K_1 is a constant and p is Poisson's ratio. In Equation (9), W is given in terms of W_{ao} and W_{bo}. Usually the cable weight is specified in terms of W_o, the weight per unit length in water of the cable at the reference state. In these cases, W_{ao} is simply given by

$$W_{ao} = W_o + W_{bo} \tag{12}$$

where W_{bc} is given by Equation (10c).

Upon using Equations (7)-(9), Equations (5) and (6) become

$$\frac{dT}{ds_o} + G_o\, f_c(e)(1 + e) + \sin\phi\left[W_{ao} - W_{bo}\, f_c^{\,2}(e)(1 + e)\right] = 0 \tag{13}$$

$$T\frac{d\phi}{ds_o} + I_o\, f_c(e)(1 + e) + \cos\phi\left[W_{ao} - W_{bo}\, f_c^{\,2}(e)(1 + e)\right] = 0 \tag{14}$$

In addition to the dependent variables T and ϕ, other dependent variables are of interest. These include x, the horizontal displacement of the cable measured from the point of attachment of the cable to the buoy, y, the vertical displacement of the cable measured from the sea surface, and the stretched cable scope s. From Figure 2, it is seen that the differential equations for x and y are given by

$$\frac{dx}{ds_o} = -\cos\phi\,(1 + e) \tag{15}$$

$$\frac{dy}{ds_o} = \sin\phi\,(1 + e) \tag{16}$$

The differential equation for s is obtained by simply rewriting Equation (7)

$$\frac{ds}{ds_o} = (1 + e) \tag{17}$$

Equations (13)-(17) along with Equation (10a) constitute six equations for the six dependent variables T, ϕ, x, y, s, and e. Upon substituting Equation (10a) into Equations (13)-(17), the result is five differential equations for the five dependent variables T, ϕ, x, y, and s.

Fluid Drag Forces

For bare cable, the normal drag force I_o is usually written as[8]

$$I_o = 1/2 \; \rho \; C_D d_o \; q \sin \phi \; |q \sin \phi| \tag{18}$$

where C_D is the drag coefficient, $q = c(y) - U_D$, and c is the ocean current which is a function of depth.

There is considerable variation among the forms proposed for the tangential drag acting on bare cable; see the survey article by Casarella and Parsons.[9] In the present study, the following form proposed by Pode[8] is used

$$G_o = fR_o \frac{(- \cos \phi)}{|\cos \phi|} \tag{19}$$

where f is an empirical constant = 0.02 for normal bare cable and $R_o = 1/2 \; \rho \; C_D d_o \; q|q|$. It should be noted that for the low velocities relative to free-floating and moored cable systems, the tangential drag is usually much smaller than the other fluid and gravity forces occurring in Equations (13) and (14). Thus, it is not necessary to use a complex form for the tangential drag. Other forms for the tangential drag can, of course, be conveniently incorporated into the formulation.

EQUATIONS FOR INTERMEDIATE BODIES

The integration of the preceding equations must be stopped when an intermediate body such as a drogue or hydrophone is reached. The tension and angle just below the body, T_b and ϕ_b, must be such that the body is in an equilibrium of forces. These forces are the tensions in the cable above and below the body and the fluid and gravity forces acting on the body. Upon solving the two equations of equilibrium for the body the following expressions for T_b and ϕ_b are obtained

$$T_b = \sqrt{(F_{Ix} + T_a \cos \phi_a)^2 + (T_a \sin \phi_a - F_{Iy})^2} \tag{20a}$$

$$\phi_b = \tan^{-1} \frac{T_a \sin \phi_a - F_{Iy}}{T_a \cos \phi_a + F_{Ix}} \tag{20b}$$

where T_a and ϕ_a are the tension and angle of the cable just above the body,
F_{Ix} is the drag force acting on the body, and
F_{Iy} is the weight in water of the body.
The drag force F_{Ix} is given by

$$F_{Ix} = 1/2\ \rho\ C_D A_I\ q|q| \tag{21}$$

where $C_D A_I$ is the drag area of the body for horizontal flow.

ITERATION SCHEMES

For chosen values of U_D and H, the integration of the equations in the preceding section proceeds until the bottom unit is reached. At this point, the equilibrium of the bottom unit is checked. As pointed out previously, if the bottom unit is in equilibrium, then the entire cable system is in equilibrium. Practically speaking, if the errors e_H and e_V are small enough to satisfy the given error criterion, the problem is considered solved. If the criterion is not satisfied, then new values must be found for U_D and/or H.

Computer program FF2E contains a preliminary scheme and two principal schemes for iterating values of U_D and H. They are as follows, in the order that they appear in the program:

(a) Preliminary scheme for U_D.

(b) Simultaneous binary scheme.

(c) Staggered binary scheme.

These schemes are described below.

PRELIMINARY SCHEME FOR U_D

In this scheme, only the value of U_D is varied. The value of H is fixed and is determined by the buoyancy required to support the weight of the entire system in water, in the absence of any current.

The scheme searches for a value of U_D which gives a value of e_H less than some small prescribed quantity e_1. The scheme does this by restricting U_D to lie between successively closer barriers. For e_H greater (less) than zero, ΔU_D is chosen so that the new value of U_D lies halfway between the present value of U_D and the most recent preceding value of U_D which is greater (less) than the present value of U_D. Basically, the changes in U_D are based on the sign given in Equation (28), which is derived in the following section.

The principal purpose of the present scheme is to provide an improved initial value of U_D for the simultaneous binary scheme. It has been found that with this initial value of U_D, the convergence of the simultaneous binary scheme is improved. Also, for certain cases of low ocean currents, the steady-state draft differs very little from the value of H fixed in the present scheme, i.e., the value of e_V associated with the final value of U_D is also small. Thus, in these cases, the present scheme presents a direct solution to the problem. Finally, as will be seen later, the present scheme essentially is the first cycle of the staggered binary scheme.

SIMULTANEOUS BINARY SCHEME

This scheme changes the values of H and U_D simultaneously.

Consider first the forces in the horizontal direction. The term $(T_H - D_B)$ appearing in the formula for e_H, given in Equation (1), represents the horizontal component of the drag forces acting on the entire cable system. It should be noted that $(T_H - D_B)$ is a complex quantity which depends on the current relative to the cable and the inclination of the cable. An approximate measure for $(T_H - D_B)$ is given by

$$T_H - D_B \sim 1/2\ \rho\ C_D A_T \left[1/2\ (c_M - U_D)^2 - 1/2\ (U_D - c_m)^2\right] \qquad (22)$$

where $C_DA_T = \left(C_DA_S + \sum_{i=1}^{N} C_{Di}\ \ell_{oi}\ d_{oi} + \sum_{i=1}^{N} C_DA_{Ii}\right)$,

C_DA_S is the drag area of the surface buoy and any underwater packages mounted on the buoy,

N is the total number of cable segments = total number of bodies,

C_{Di}, ℓ_{oi}, and d_{oi} are respectively the drag coefficient, reference length, and reference diameter of the ith cable segment,

C_DA_{Ii} is the drag area of the ith body,

c_M is the maximum value of the current, and

c_m is the minimum value of the current.

It can be seen from the above definition for C_DA_T that it is a measure of the drag area of the entire cable system. Noting that T_{BH} is simply a constant in Equation (1), it can be seen that e_H depends directly on $(T_H - D_B)$. In particular, it can be seen that if e_H is greater (less) than zero, the change in $(T_H - D_B)$, $\Delta(T_H - D_B)$, should be less (greater) than zero. Thus, the error e_H may be related to $\Delta(T_H - D_B)$ as follows

$$e_H = - \Delta(T_H - D_B) \sim - 1/2\ \rho\ C_DA_T \left[(c_M - U_D)(- \Delta U_D) - (U_D - c_m)\ \Delta U_D\right]$$

$$= \Delta U_D\ \rho\ C_DA_T\ 1/2\ (c_M - c_m) = \Delta U_D M_U \tag{23}$$

where $M_U = \rho\ C_DA_T\ 1/2\ (c_M - c_m)$. An expression for ΔU_D in terms of e_H is then given by

$$\Delta U_D \sim \frac{e_H}{M_U} \tag{24}$$

Consider now the vertical forces. By noting that the quantities W_B and T_{BV} are constants in the formula for e_V, given in Equation (2), it can be seen that e_V depends directly on T_V. In particular, it can be seen that if e_V is greater (less) than zero, the change in T_V, ΔT_V, should be less (greater) than zero. Provided that changes in the vertical component of tension just below the buoy are propagated without a reversal in sign

to the bottom of the cable, e_V may be related to changes in the buoyancy of the surface buoy B_S. The buoyancy force B_S is given by

$$B_S = \rho g \frac{\pi d_S^{\,2}}{4} H = M_V H \tag{25}$$

where d_S is the diameter of the surface buoy and $M_V = \rho g \; \pi d_S^{\,2}/4$. From the preceding discussion, e_V may be related to ΔB_S, the change in B_S, as follows

$$e_V = - \Delta T_V \sim - \Delta B_S = - M_V \, \Delta H \tag{26}$$

An expression for ΔH in terms of e_V is then given by

$$\Delta H \sim \frac{-e_V}{M_V} \tag{27}$$

A number of proportionality factors have been tested for Equations (24) and (27). No factors were found which cause the scheme to converge to the equilibrium values of H and U_D for all cases. The optimum definitions for ΔU_D and ΔH were found to be of the following form

$$\Delta U_D = \frac{\delta \cdot \delta_H \, |e_H| \, e_H}{E \cdot M_U} \tag{28}$$

$$\Delta H = \frac{-\delta \cdot |e_V| \, e_V}{E \cdot M_V} \tag{29}$$

where δ is a positive proportionality quantity, $0 < \delta \leq 1$ and δ_H is a positive quantity. As discussed in the following paragraph, the quantity δ serves the purpose of reducing the changes in U_D and H if these changes lead to a resultant error which is larger than the preceding resultant error. The quantity δ_H is initially set equal to 1. If the changes in e_H occur too slowly (fast), then δ_H is increased (decreased). The ratios $|e_H|/E$ and $|e_V|/E$ which occur in Equations (28) and (29), respectively,

basically tell the scheme to concentrate on changing $U_D(H)$ if the predominant error is $e_H(e_V)$. It was found that the presence of these ratios increases the range of convergence of the scheme.

The scheme starts by choosing the initial values for H and U_D to be the final values of these quantities from the preliminary scheme and calculating the cable configuration from these initial values. The quantities ΔU_D and ΔH are computed from the resulting error values e_V, e_H, and E by using an initial value of δ equal to 1. These values of ΔU_D and ΔH are added respectively to U_D and H, resulting in new values of H and U_D, H' and U_D'. The cable configuration is again calculated using the new values H' and U_D'. New values of error are obtained: e_H', e_V', and E'. If E' < E, then δ remains at its initial value, and the new values H' and U_D' are retained. The quantities ΔU_D and ΔH are obtained from e_H', e_V', and E'. If, however, E' > E, then the new values e_H', e_V', E', H', and U_D' are rejected. The quantities ΔU_D and ΔH are obtained from the old values of e_H, e_V, and E using a smaller value of δ. The quantity δ is continually reduced until:

(a) E' < E,

(b) e_H' simultaneously satisfies the following two conditions:

$$(e_H'/e_H) > 1 \text{ and}$$

$$|e_H'| > |e_V'|, \text{ or}$$

(c) δ is lower than a prescribed lower bound. This lower bound is set equal to 0.2 for the first ten iterations of the scheme and 0.05 for subsequent iterations.

In any of the above three cases, δ is reset equal to its initial value of 1, and ΔU_D and ΔH are computed by using the new error values e_H', e_V', and E'. The resulting values of ΔU_D and ΔH are added to U_D' and H', respectively. The scheme continues in the above manner until the errors e_H and e_V are sufficiently small so that the given error criterion is satisfied.

In one of the earlier schemes tested, the quantity δ was allowed to decrease indefinitely until E' < E. In other words, options (b) and (c) of the preceding paragraph were not present. Furthermore, δ was never

reset equal to its original value, but either remained the same or decreased (according to whether E' was less than or greater than E) as the iteration proceeded. This scheme was similar to one proposed by Skop and O'Hara,[3] who dealt with the problem of subsurface floats moored by an array of cables. It was soon found that this scheme did not converge in many cases. It was found that at certain steps of the iteration, the new error E' could never be made smaller than E, no matter how small δ was made. It was found that this was due to interaction effects whereby changes in H have large effects on e_H and/or changes in U_D have large effects on e_V. It should be noted that interaction effects are not represented in Equations (28) and (29).

It was found that these interaction effects arise in cases where the current differential between the top and bottom of the cable system is fairly large. In these cases, the normal fluid drag forces acting on the cable, given in Equation (18), become significantly large. The interaction effects arise largely due to the strong dependence of the drag forces on cable geometry as shown in Equation (18). Consider, for example, the case where only U_D or H is changed. For a given change, the geometry of the entire cable may be changed significantly, and the resulting changes in the normal drag forces may have significant components in both directions.

Skop and O'Hara[3] prove that their scheme converges for moored cable arrays. It should be noted, however, that the proof depends on the assumption that all of the forces acting on the cable are unaffected by changes in the geometry of the cable. Thus, their work should be viewed with some caution, particularly for cases of high ocean currents where the geometry dependent fluid drag forces become significant.

The results of Reference 3 do suggest, however, that the convergence of the present scheme may be improved by restricting the cable shape to lie between certain bounds. An auxiliary scheme has been incorporated into the program which changes U_D by small amounts whenever the angle ϕ exceeds or becomes less than certain prescribed bounds. This auxiliary scheme has been found to improve the convergence of the main scheme. It should be noted that the bounds on ϕ can neither be too close nor too far apart. If the bounds on ϕ are too far apart, then it is clear that the scheme loses its effectiveness. On the other hand, excessively close bounds may mean many changes of U_D, thus increasing computer time requirements. Also, the

equilibrium cable configuration may lie outside the bounds if they are too restrictive. It has been found that the optimum lower and upper bounds for ϕ are approximately 0 degrees and 125 degrees, respectively.

As mentioned previously, this scheme does not converge for all cases. The limits of convergence seem to depend most strongly on the current differential between the top and bottom of the cable system, and on the length of the cable. Based on many hundreds of runs already made with the program, it seems that the scheme converges for the large majority of cases where the current differential is less than 3.0 knots. In these cases, less than approximately twenty iterations are usually required to obtain values of H and U_D which yield values of the resultant error E less than, say, 0.05 pounds. For some cases of high current differentials, the scheme cannot minimize E beyond a certain amount, ranging from a few tenths of a pound to several pounds. In these cases, the computer program switches to the staggered binary scheme.

STAGGERED BINARY SCHEME

Description of Scheme

The preliminary scheme constitutes the first cycle of the staggered binary scheme. Basically, the scheme changes H and U_D one at a time, thus avoiding the interaction effects mentioned above. The changes in H and U_D are made in accordance with the signs indicated in Equations (29) and (28), respectively. First, an initial value of H is chosen to be the value required to support the weight of the entire system in water, in the absence of any current. Then, keeping the value of H constant, the scheme searches for a value of U_D which gives a value of e_H less than e_1. The procedure for doing this has already been discussed in the section dealing with the preliminary scheme.

When a value of U_D has been found which gives rise to $e_H < e_1$, the iteration for U_D stops. The scheme calculates a new value for H based on the value of e_{VO}, the value of e_V associated with $e_H < e_1$, as follows

$$\Delta H = \frac{-\delta_V e_{VO}}{M_V} \tag{30}$$

where δ_V is a positive number and M_V is defined in Equation (25).

It may be noted that Equation (30) differs from Equation (29) in that the quantities $|e_V|/E$ and δ which occur in Equation (29) have been replaced by 1 and δ_V, respectively. For the case of e_{VO}, where $e_H < e_1$, the ratio $|e_V|/E$ is nearly equal to 1. The quantity δ_V is initially chosen to be 0.6. Should the convergence to the equilibrium value of H be excessively slow, δ_V is increased. Similarly, δ_V is decreased if the changes in H cause excessively large changes in e_{VO}. It should be noted that H is also restricted to be between successively closer barriers. For e_{VO} greater (less) than zero, the new value of H is restricted to be greater (less) than the last preceding value of H which gives rise to an error e_{VO} less (greater) than zero. Should the value of ΔH calculated in Equation (30) lead to a new value of H which goes beyond the preceding barrier, the new value of H is adjusted so that it lies between the present value of H which gives rise to the given value of e_{VO} and the barrier. Keeping this new value of H constant, the scheme again searches for a value of U_D which gives rise to $e_H < e_1$. The scheme proceeds in the above manner until the error quantities e_H and e_V are small enough to satisfy the given error criterion

Discussion of Convergence

This scheme converges over a wider range of cable and current parameters than the simultaneous binary scheme. It is, however, slower, usually requiring at least sixty iterations to arrive at suitably small values of e_H and e_V. Based on many hundreds of computer runs made thus far, it seems that the scheme converges for nearly all cases where the surface current is less than approximately 5.0 knots. It should be noted that surface currents seldom exceed 5 knots (see, for example, Reference 10).

For some cases of long cables floating in the presence of surface currents in excess of 5 knots, the scheme may fail to converge. In these cases, where the geometry dependent fluid drag forces dominate, the scheme cannot minimize the error E beyond a certain amount ranging from a few

tenths of a pound to several pounds. The computer results for these cases show that the cable configuration and the error quantities e_H and e_V are very sensitive to changes in H and U_D. As an example, consider the particular case of a 2,000-ft cable floating in the presence of a surface current of 8 knots. For this case, the computer results showed that at H = 0.86318 ft, a change in H in the sixth or higher decimal place produced a change in e_{VO} from -6.2 lb to 1.1 lb. This large change of e_{VO} from negative to positive values is typical of the behavior of the scheme when it fails to converge.

The preceding results were obtained for the case when the differential Equations (13)-(17) were integrated to an accuracy of 0.01 percent. In view of the extreme sensitivity of e_{VO} to changes in H, it was decided to investigate this particular case further by increasing the accuracy of the integration of the differential equations. With accuracies up to and including 0.0001 percent, the large changes of e_{VO} with changes in H in the sixth or higher decimal place around $e_{VO} = 0$ continued to occur. The values of H giving rise to these large changes in e_{VO} changed somewhat with the accuracies. For example, with accuracies of 0.001 percent and 0.0001 percent, the large changes in e_{VO} from negative to positive values occurred respectively at H = 0.86199 ft and H = 0.86066 ft. The investigation had to be terminated when an attempt was made to integrate the differential equations with an accuracy of 0.00001 percent. Computer time requirements became prohibitively large.

The preceding discussion shows the nature of the physical and/or numerical instability for cases of unusually severe current profiles and sufficiently long cables where the cable geometry dependent fluid drag forces dominate. In order to thoroughly investigate the nature of this instability, and to determine the solution, computer time requirements beyond the scope of this study would be required. The preceding results do tentatively indicate that the equilibrium value of H is localized to a small neighborhood by the scheme. In most cases where the scheme fails to converge, the minimum resultant error does not exceed a few pounds. It is expected that in most engineering applications involving cables a few thousand feet or longer, an error of one or two pounds can be tolerated.

DISCUSSION OF UNIQUENESS

The uniqueness of the numerical solutions has not been investigated in the present study. One way to study this question is to incorporate a grid scheme into the computer program whereby H and U_D are each assigned a given number of values over prescribed intervals. The configuration of the cable and the resultant error E are then calculated for each possible pair of values of H and U_D. If E is equal to zero for two or more different pairs of values, then the solutions are nonunique. Based on the sensitivity of E to changes in H and U_D, observed from the many runs using program FF2E, it seems that in most cases a minimum of 50 to 100 values must be prescribed for both H and U_D, around their respective equilibrium values predicted by program FF2E, in order to discover the existence of any additional solutions which may exist. This means that for a given case, the configuration of the cable must be calculated 2,500 to 10,000 times. The execution time on the CDC 6700 for a single calculation of the configuration of the cable is of the order of one second. Thus, several hours of computer time probably would be required in order to investigate uniqueness for one case. A systematic investigation would require the consideration of a large number of cases. The computer times required were beyond the scope of the present study.

FORTRAN IV COMPUTER PROGRAM

Based on the preceding analysis, a FORTRAN IV computer program, FF2E, has been written for the system shown in Figure 1. The program is compatible with the CDC 6700 computer currently being used at NSRDC. The program consists of a main program and four subroutines. The main program reads in the parameters for the surface buoy, cable segments, intermediate bodies, and bottom weight, as well as the wind speed and current profile. The main program also prints out the input data.

It is worthwhile to make several comments about the data which are read in. The program reads in data for a cylindrical surface buoy. However, by reading in suitable values for the length, diameter, and drag coefficient, other surface buoy shapes may be approximated. Also, as previously mentioned, the program assumes that the cable is linearly

elastic by reading in, for each cable segment, the constant quantity K_1, given in Equation (11a). It is felt that this approximation is sufficiently accurate for the large majority of engineering applications. If one wishes to accurately model nonlinear tension-strain behavior, the subroutine ELAS which is present in program MR3E[2] may be conveniently incorporated into the present program. It should be noted that in the above program the differential equations are integrated only once. As pointed out earlier in the section on iteration schemes, the differential equations may have to be integrated 100 times or more in the present program before a solution is achieved. Thus, the incorporation of the subroutine ELAS will increase computer time requirements for the present program. Finally, it should be noted that in some free-floating cable systems, the lower intermediate bodies form an array of hydrophones. In this case, the reader simply reads in the number of lower intermediate bodies, exclusive of the bottom weight, which form the array. As pointed out later in the present section, the computer program prints out additional information about the configuration of the array.

After reading in the input data, the program searches for the equilibrium values of U_D and H. This is done by the iteration schemes, described previously, contained in the subroutine STEADY. For given values of H and U_D, the differential Equations (13)-(17) are integrated down the cable. The differential equations are defined in the subroutine DAUX and integrated by the subroutine KUTMER, which uses the Kutta-Merson method for solving systems of first-order differential equations. The differential equations are integrated to an accuracy of 0.01 percent. By simply changing one or two statements in the program, any accuracy may be prescribed. As mentioned earlier, computer time requirements increase with increasing accuracy. The proper value of the current is given by the subroutine CUR which linearly interpolates the input current profile points. The calculation of Equations (20a)-(21) in the case of intermediate bodies is performed by the subroutine STEADY.

When the bottom of the cable is reached, the errors e_V and e_H are calculated and are tested to see if they satisfy the inequalities given in Equations (4a) and (4b). If both inequalities are not simultaneously satisfied, new values must be found for H and/or U_D. Before doing so,

the subroutine STEADY prints out the iteration number, the values of U_D, H, e_V, e_H, D_B, δ (used in Equations (28) and (29)), and the tensions in the cable just below the surface buoy and just above the bottom weight.

If both of the inequalities (4a) and (4b) are simultaneously satisfied, the program stops iterating for further values of H and U_D and goes on to print the following information:

1. System drift velocity U_D and buoy draft H.
2. Vertical and horizontal components of the tension in the cable just below the surface buoy.
3. Variables at points along the cable, as functions of the reference cable scope s_o measured from the surface buoy:

   s, stretched cable scope,

   x, horizontal displacement relative to buoy bottom,

   y, vertical displacement relative to ocean surface,

   T, tension in the cable,

   ϕ, angle measured from the horizontal, and

   ϕ_V, angle measured from the vertical.
4. Fluid and gravity forces D_B and W_B acting on the bottom unit.
5. Vertical and horizontal components of the tension just above the bottom unit.

In the event that there are two or more lower hydrophones, exclusive of the bottom weight, the program prints out Θ, the angle with the vertical of a chord connecting the top of the array with the bottom weight, and k, the maximum perpendicular distance between the chord and array. The quantities Θ and k are shown in Figure 3. It should be noted that the coordinate system (x_r, y_r) shown in Figure 3 is defined such that the bottom weight is the origin of the coordinate system.

After printing out the above information, the program goes on to a new case.

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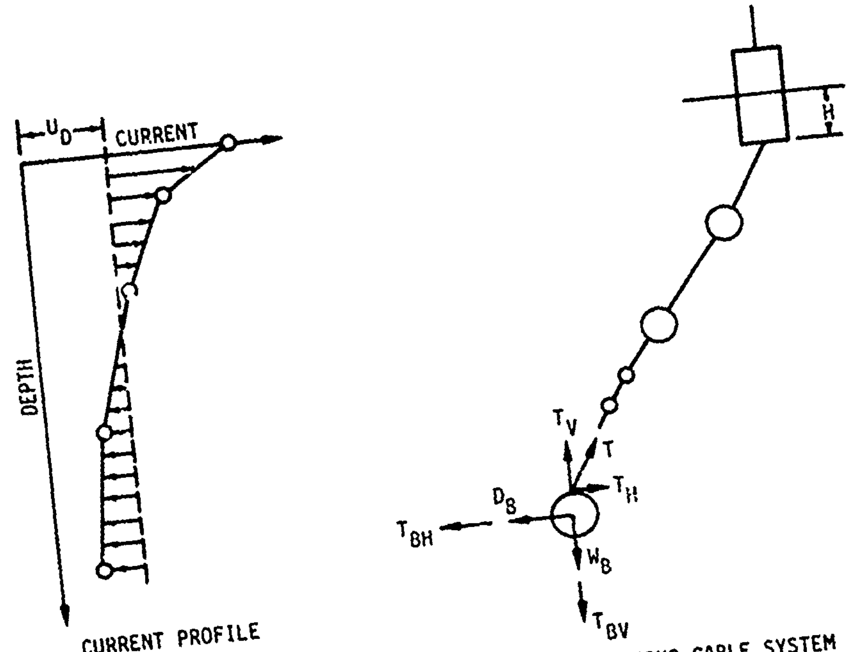

Figure 1 - Description of Free-Floating Cable System in a Current Profile

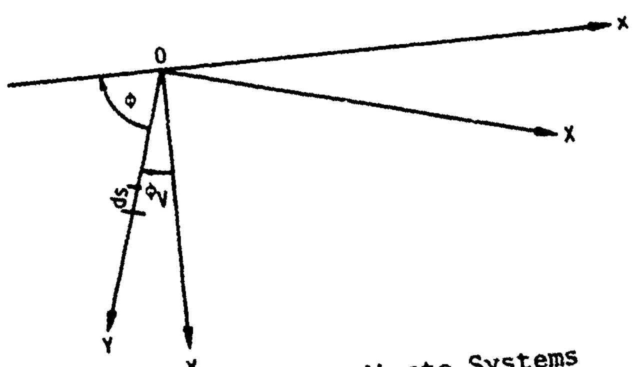

Figure 2 - Coordinate Systems

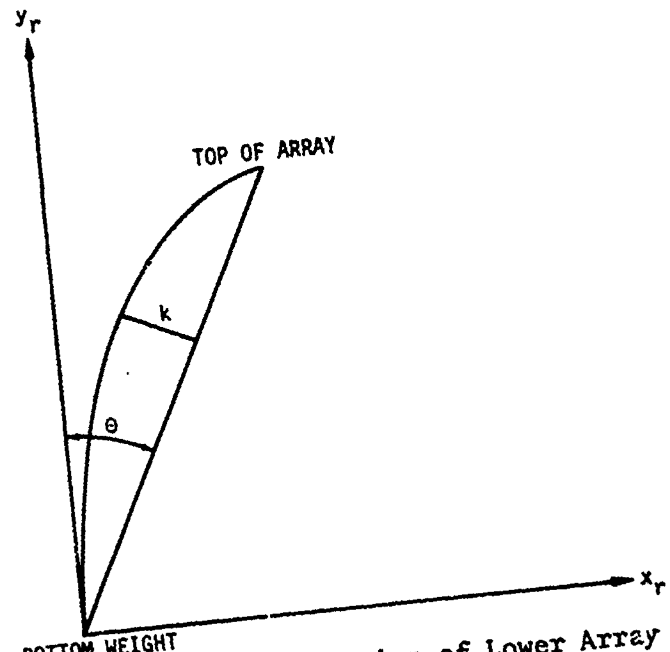

Figure 3 - Configuration of Lower Array

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